647510 **BSCabsCC6020** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
April/May-2022 (OLD COURSE)
MATHEMATICAL ANALYSIS-2 AND ABSTRACT ALGEBRA -2(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

QUE. 1 (A) Answer any one out of two. **[7]**

(1) Show that compact subset of metric space is closed.

(2) Show that continuous image of connected set is Connected.

(B) Answer any one out of two. **[4]**

(1) X is metric space and E_1 and E_2 be two connected subsets of X and $E_1 \cap E_2 \neq \phi$ then show that $E_1 \cup E_2$ is Connected.

(2) Show that set of all even integers is countable set.

(C) Answer any one out of two. **[3]**

(1) Check whether R-{0} is connected. where R is set of all real numbers.

(2) Check whether the following subsets of R are compact.

(i) {-1,0,3,4,12} (ii) $[1,3] \cap (3,5)$ (iii) $\{\frac{1}{n} / n \in N\}$

QUE. 2 (A) Answer any one out of two. **[7]**

(1) State and prove first shifting theorem and find $L[\sin^3(2t)]$.

(2) State and prove change the scale property. Using this Property Find $L[t^3e^{-3t}]$.

(B) Answer any one out of two. **[4]**

(1) Find : $L^{-1}[\frac{1}{(s^2+a^2)^2}]$

(2) Find Laplace transform of

$f(t) = \sin(t)$ where $0< t< \pi$

$= 0$ where $t>\pi$

(C) Answer any one out of two. **[3]**

(1) Find : $L[x^2\sin(hx)]$.

(2) Find : $L^{-1}[\frac{s+7}{s^2+2s+2}]$.

QUE. 3 (A) Answer any one out of two. **[7]**

(1) Prove that if $L\{f(t)\}=\bar{f}(s)$ then $L[\int_0^t f(u)du]=\frac{1}{s}\bar{f}(s)$. Using this result Find

$L^{-1}[\frac{s+1}{(s^2+4)s^2}]$.

(2) if $L\{f(t)\}=\bar{f}(s)$ and $\frac{f(t)}{t}$ has laplace transform then prove

That $L\{\frac{f(t)}{t}\}=\int_s^\infty \bar{f}(s)ds$. Using this result Find $L[\frac{\sin(t)}{t}]$.

(B) Answer any one out of two. **[4]**

(1) Using convolution theorem Find $L^{-1}[\frac{s}{(s+1)(s^2+1)}]$.

(2) Find : $L[e^{-4t}\int_0^t t\sin(3t)dt]$.

(C) Answer any one out of two. **[3]**

(1) Find $L[t^n]$ Where n is Non negative integer.

(2) Find : $L^{-1}[\frac{1}{s(s-1)}]$.

QUE. 4 (A) Answer any one out of two. **[7]**

(1) Prove that if $H \leq G$ then $\phi(H) \leq G'$ where (G,*) and (G',*') be two groups and $\phi : G \to G'$ is homomorphism.

(2) If P is prime number then show that Z_p is field.

(B) Answer any one out of two. **[4]**

(1) Prove that characteristic of integral domain is either Zero or prime number.

(2) For Non zero polynomial $f(x),g(x) \in D[X]$ show that $[f(x).g(x)]=[f(x)]+[g(x)]$.

(C) Answer any one out of two. **[3]**

(1) Illustrate with example of subring of ring which is right ideal but not left ideal.

(2) Find zeros of $f(x)=4x^2+3x+2 \in Z_5[X]$.

QUE. 5 (A) Answer any one out of two. **[7]**

(1) State and prove division algorithm theorem of polynomial.

(2) Show that integral domain D[X] is not field.

(B) Answer any one out of two. **[4]**

(1) Show that in $Z_5[X]$, $x^4+3x^3+2x+4=(x-1)^3(x+1)$.

(2) Check whether $f(x)=x^3-2x^2+9$ is irreducible over Q.

(C) Answer any one out of two. **[3]**

(1) Find G.C.D. of $f(x),g(x) \in Z_5[X]$. Where $f(x)=x^3+3x^2+3x+3$ and $g(x)=4x^3+2x^2+2x+2$.

(2) if $a=a_1+a_2i+a_3j+a_4k$ is non-zero quaternion number then find a^{-1}. Also check whether quaternion set is field.

****